ending literature of controversy. In the contrast between Galileo's heroic stand when he tried to reform the cosmological basis of orthodox theology and his humbled, kneeling surrender when he disavowed his Copernicanism, we may sense the tremendous forces attendant on the birth of modern science. And we may catch a glimpse of the spirit of this great man as we think of him, after his trial and condemnation, living under a kind of house-arrest or surveillance as Milton saw him in Arcetri, completing his greatest scientific work, *Discourses and Demonstrations Concerning Two New Sciences.* This book was the base from which the next generation of scientists would begin the great exploration of the dynamical principles of a sun-centered universe.

CHAPTER 6

Kepler's Celestial Music

Since Greek times scientists have insisted that Nature is simple. A familiar maxim of Aristotle is, "Nature does nothing in vain, nothing superfluous." Another expression of this philosophy has come down to us from a fourteenth-century English monk and scholar, William of Occam. Known as his "law of parsimony" or "Occam's razor" (perhaps for its ruthless cutting away of the superfluous), it maintains, "Entities are not to be multiplied without necessity." "It is vain to do with more what can be done with fewer" perhaps sums up this attitude. As Newton put it, in the *Principia,* "Nature does nothing in vain, and more causes are in vain when fewer suffice." The reason is that "Nature is simple and does not indulge in the luxury of superfluous causes."

We have seen Galileo assume a principle of simplicity in his approach to the problem of accelerated motion, and the literature of modern physical science suggests countless other examples. Indeed, present-day physics is in distress, or at least in an uneasy state, because the recently discovered nuclear "fundamental particles" exhibit a stubborn disinclination to recognize simple laws. Only a few decades ago physicists complacently assumed that the proton and the electron were the only "fundamental particles" they needed to explain the atom. But now one "fundamental particle" after another has crept into the ranks until it appears that there may possibly be as many of them as there are chemical elements. Confronted with this bewildering array, the average physicist is tempted to echo Alfonso the Wise and bemoan the fact that he was not consulted first.

Anyone who examines Fig. 14 (pp. 46-47) will see at once that neither the Ptolemaic nor the Copernican system was, in any sense of the word, "simple." Today we know why these systems lacked simplicity: restricting celestial motions to circles introduced many otherwise unnecessary curves and centers of motion. If astronomers had used some other curves, notably the ellipse, a smaller number of them would have done the job better. It was one of Kepler's great contributions to astronomy to have found this truth.

THE ELLIPSE AND THE KEPLERIAN UNIVERSE

The ellipse enables us to center the solar system on the true sun rather than some "mean sun" or the center of the earth's orbit as Copernicus did. Thus the Keplerian system displays a universe of stars fixed in space, a fixed sun, and a *single* ellipse for the orbit of each planet, with an additional one for the moon. In actual fact, most of these ellipses, except for Mercury's orbit, look so much like circles that at first glance the Keplerian system seems to be the simplified Copernican system shown on page 47 of Chapter 3: one circle for each planet as it moves around the sun, and another for the moon.

An ellipse (Fig. 22) is not as "simple" a curve as a circle, as will be seen. To draw an ellipse (Fig. 22A), stick two pins or thumbtacks into a board, and to them tie the ends of a piece of thread. Now draw the curve by moving a pencil within the loop of thread so that the thread always remains taut. From the method of drawing the ellipse, the following defining condition is apparent: every point P on the ellipse has the property that the sum of the distances from it to two other points F_1 and F_2, known as the *foci*, is constant. (The sum is equal to the length of the string.) For any pair of foci, the chosen length of the string determines the size and shape of the ellipse, which may also be varied by using one string-length and placing the pins near to, or far from, each other. Thus an ellipse may have a shape (Fig. 22B) with more or less the proportions of an egg, a cigar, or a needle, or may be almost round and like a circle. But unlike the true egg, cigar, or needle, the ellipse must always be symmetrical (Fig. 23) with respect to the axes, one of which (the major axis) is a line

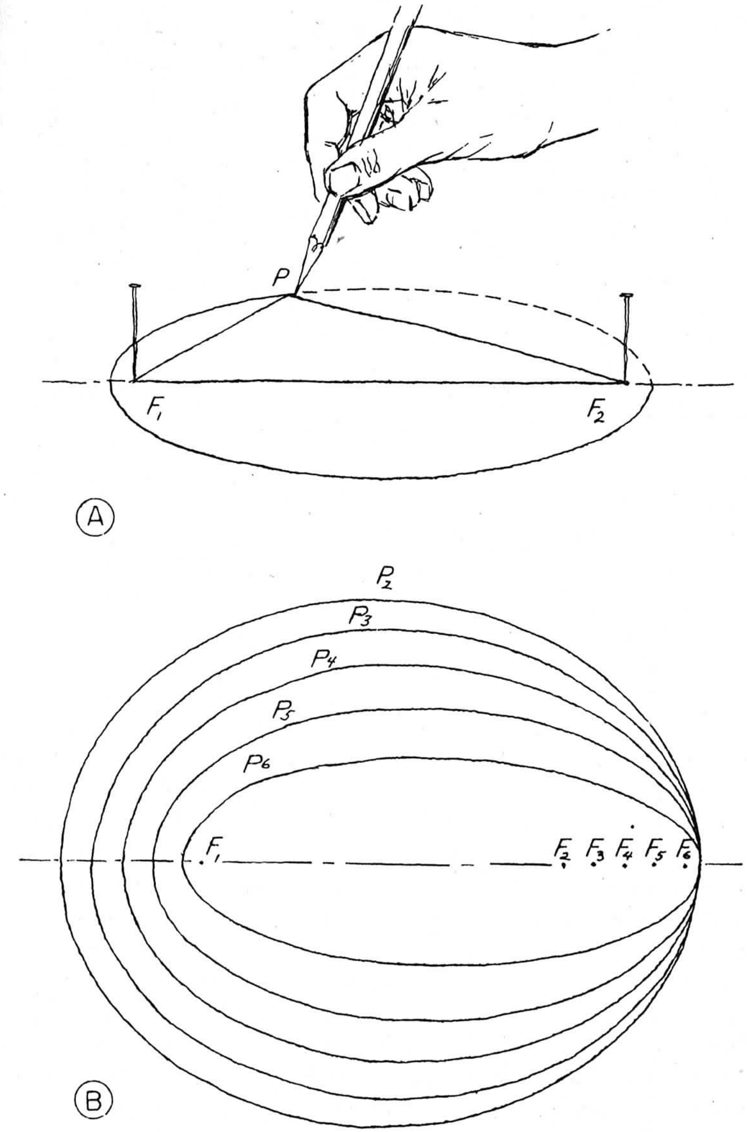

FIG. 22. The ellipse, drawn in the manner shown in (A), can have all the shapes shown in (B) if you use the same string but vary the distance between the pins, as at F_2, F_3, F_4, etc.

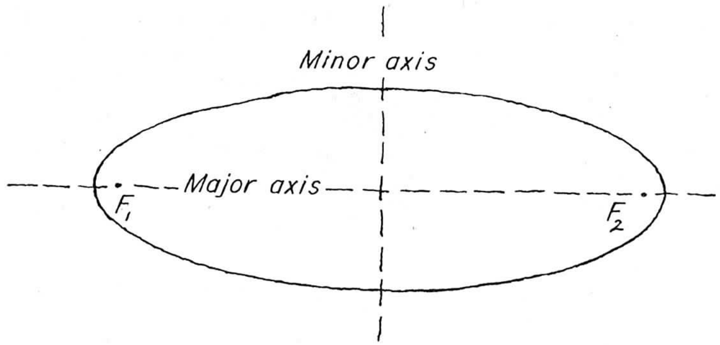

FIG. 23. The ellipse is always symmetrical with respect to its major and minor axes.

drawn across the ellipse through the foci and the other (the minor axis) a line drawn across the ellipse along the perpendicular bisector of the major axis. If the two foci are allowed to coincide, the ellipse becomes a circle; another way of saying this is that the circle is a "degenerate" form of an ellipse.

The properties of the ellipse were described in antiquity by Apollonius of Perga, the Greek geometer who inaugurated the scheme of epicycles used in Ptolemaic astronomy. Apollonius showed that the ellipse, the parabola (the path of a projectile according to Galilean mechanics), the circle, and another curve called the hyperbola may be formed (Fig. 24) by passing planes at different inclinations through a right cone, or a cone of revolution. But until the time of Kepler and Galileo, no one had ever shown that the conic sections occur in the natural phenomena of motion.

In this work we shall not discuss the stages whereby Johannes Kepler came to make his discoveries. Not that the subject is devoid of interest. Far from it! But at present we are concerned with the rise of a new physics, as it was related to the writings of antiquity, the Middle Ages, the Renaissance and the seventeenth century. Aristotle's books were read widely, and so were the writings of Galileo and Newton. Men studied Ptolemy's *Almagest*

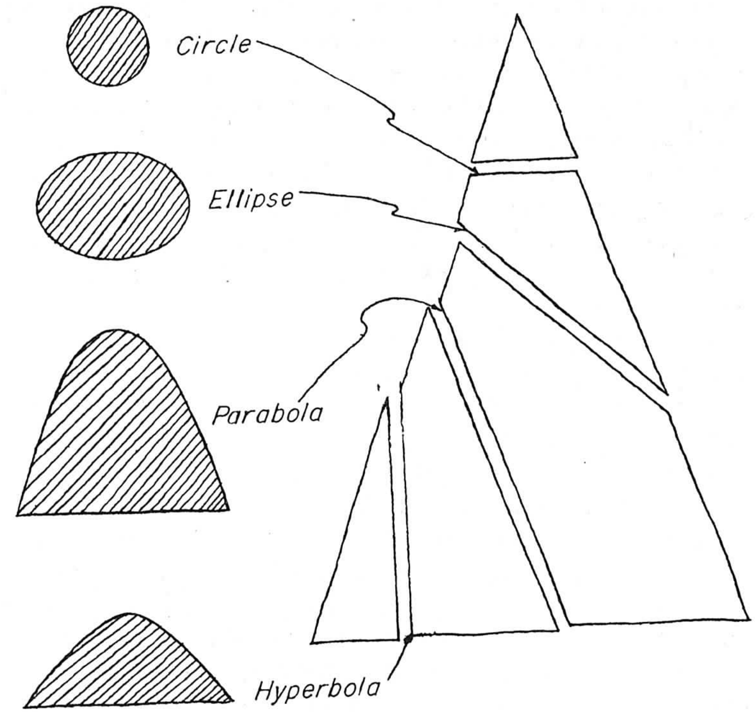

FIG. 24. The conic sections are obtained by cutting a cone in ways shown. Note that the circle is cut parallel to the base of the cone, the parabola parallel to one side.

and Copernicus's *De revolutionibus* carefully. But Kepler's writings were not so generally read. Newton, for example, knew the works of Galileo, but he apparently did not read Kepler's astronomical works. He acquired his knowledge of Kepler's laws at second hand, from T. Streete's handbook of astronomy and V. Wing's textbook. Even today the major works of Kepler are not available in complete English, French, or Italian translations.

This neglect of Kepler's texts is not hard to understand. The language and style are of unimaginable difficulty and prolixity, which, in contrast with the clarity and vigor of Galileo's every word, seem formidable beyond endurance. This is to be expected, for writing reflects the personality of the author. Kepler was a tortured mystic, who stumbled onto his great discoveries in a weird groping that has led one of his biographers* to call him a "sleepwalker." Trying to prove one thing, he discovered another, and in his calculations he made some major errors that canceled each other out. He was utterly unlike Galileo and Newton; never could their purposeful quests for truth conceivably merit the description of sleepwalking. Kepler, who wrote sketches of himself, said that he became a Copernican as a student and that "There were three things in particular, namely, the number, distances and motions of the heavenly bodies, as to which I [Kepler] searched zealously for reasons why they were as they were and not otherwise." About the sun-centered system of Copernicus, Kepler at another time wrote: "I certainly know that I owe it this duty: that since I have attested it as true in my deepest soul, and since I contemplate its beauty with incredible and ravishing delight, I should also publicly defend it to my readers with all the force at my command." But it was not enough to defend the system; he set out to devote his whole life to finding a law or set of laws that would show how the system held together, why the planets had the particular orbits in which they are found, and why they move as they do.

The first installment in this program, published in 1596, when Kepler was twenty-five years old, was entitled *Forerunner of the Dissertations on the Universe, Containing the Mystery of the Universe.* In this book Kepler announced what he considered a great discovery concerning the distances of the planets from the sun. This discovery shows us how rooted Kepler was in the Platonic-Pythagorean tradition, how he sought to find regularities in nature associated with the regularities of mathematics. The Greek geometers had discovered that there are five "regular solids," which are shown in Fig. 25. In the Copernican system there are six planets: Mercury, Venus, Earth, Mars, Jupiter, Saturn. Hence

*Arthur Koestler, *The Sleepwalkers* (London: Hutchinson & Co., 1959).

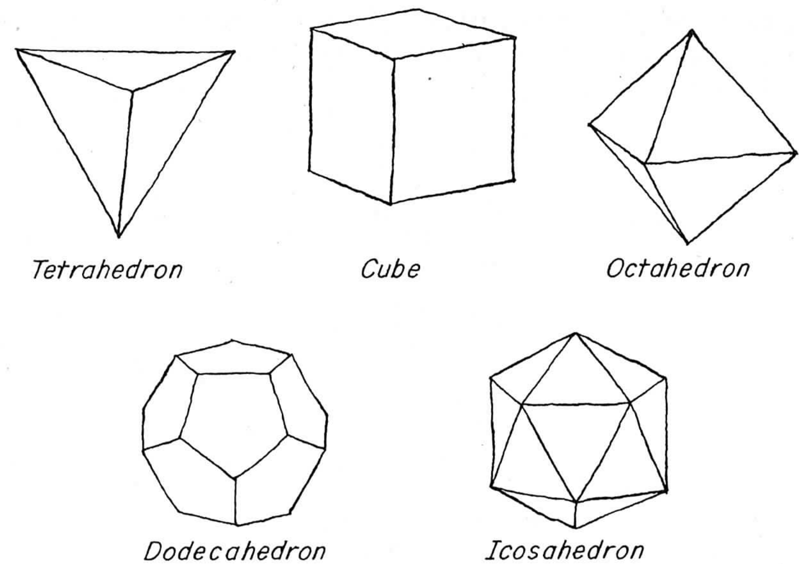

FIG. 25. The "regular" polyhedra. The tetrahedron has four faces, each an equilateral triangle. The cube has six faces, each a square. The octahedron has eight faces, each an equilateral triangle. Each of the dodecahedron's twelve faces is an equilateral pentagon. The twenty faces of the icosahedron are all equilateral triangles.

it occurred to Kepler that five regular solids might separate six planetary orbits.

He started with the simplest of these solids, the cube. A cube can be circumscribed by one and only one sphere, just as one and only one sphere can be inscribed in a cube. Hence we may have a cube that is circumscribed by sphere No. 1 and contains sphere No. 2. This sphere No. 2 just contains the next regular solid, the tetrahedron, which in turn contains sphere No. 3. This sphere No. 3 contains the dodecahedron, which in turn contains sphere No. 4. Now it happens that in this scheme the radii of the successive spheres are in more or less the same proportion as the mean distances of the planets in the Copernican system except for

Jupiter—which isn't surprising, said Kepler, considering how far Jupiter is from the sun. The first Keplerian scheme (Fig. 26), then, was this:

Sphere of Saturn
Cube
Sphere of Jupiter
Tetrahedron
Sphere of Mars
Dodecahedron
Sphere of Earth
Icosahedron
Sphere of Venus
Octahedron
Sphere of Mercury.

"I undertake," he said, "to prove that God, in the creation of this mobile universe and the arrangement of the heavens, had in view the five regular bodies of geometry celebrated since the days of Pythagoras and Plato, and that He has accommodated to their nature, the number of the heavens, their proportions, and the relations of their movements." Even though this book fell short of unqualified success, it established Kepler's reputation as a clever mathematician and as a man who really knew something about astronomy. On the basis of this performance, Tycho Brahe offered him a job.

Tycho Brahe (1546–1601) has been said to have been the reformer of astronomical observation. Using huge and well-constructed instruments, he had so increased the accuracy of naked-eye determinations of planetary positions and of the locations of the stars relative to one another that it was clear that neither the system of Ptolemy nor that of Copernicus could truly predict the celestial appearances. Furthermore, in contrast to earlier astronomers, Tycho did not merely observe the planets now and then to provide factors for a theory or to check such a theory; instead he observed a planet whenever it was visible, night after night. When Kepler eventually became Tycho's successor, he inherited the largest and most accurate collection of planetary observations—notably for the planet

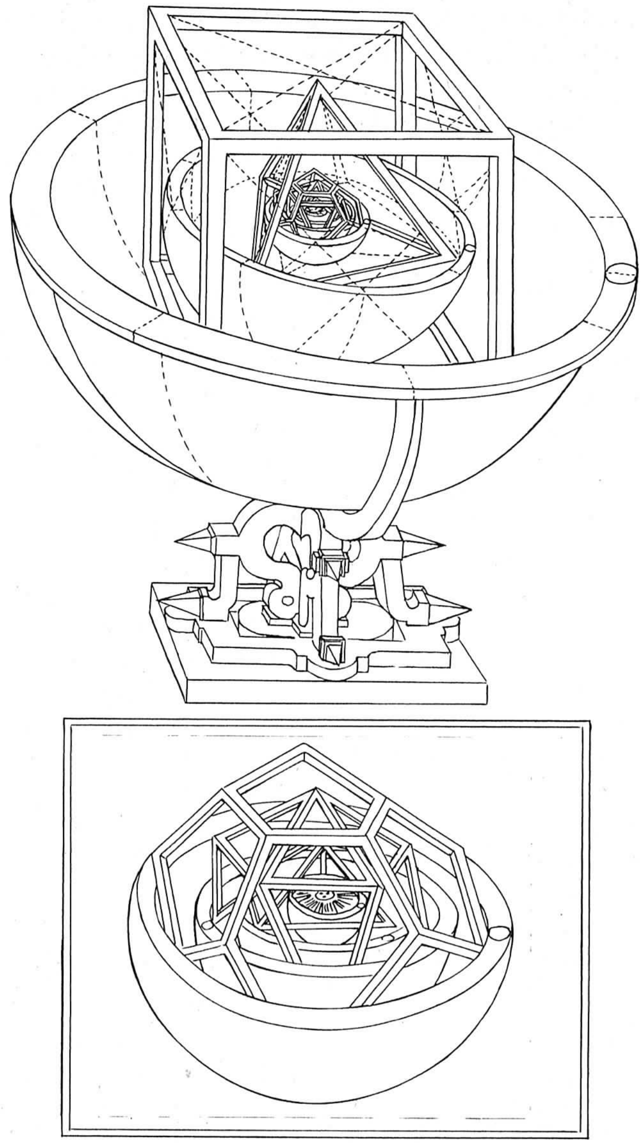

Fig. 26. Kepler's model of the universe. This weird contraption, consisting of the five regular solids fitted together, was dearer to his heart than the three laws on which his fame rests. From his book of 1596.

Mars—that had ever been assembled. Tycho, it may be recalled, believed in neither the Ptolemaic nor the Copernican system but had advanced a geocentric system of his own devising. Kepler, faithful to a promise he had made to Tycho, tried to fit Tycho's data on the planet Mars into the Tychonian system. He failed, as he failed also to fit the data into the Copernican system. But twenty-five years of labor did produce a new and improved theory of the solar system.

Kepler presented his first major results in a work entitled *A New Astronomy . . . Presented in Commentaries on the Motions of Mars,* published in 1609,* the year in which Galileo first pointed his telescope skyward. Kepler had made seventy different trials of putting the data obtained by Tycho into the Copernican epicycles and the Tychonian circles but always failed. Evidently it was necessary to give up all the accepted methods of computing planetary orbits or to reject Tycho's observations as being inaccurate. Kepler's failure may not appear as miserable as he seemed to think. After calculating eccentrics, epicycles, and equants in ingenious combinations, he was able to obtain an agreement between theoretical predictions and the observations of Tycho that was off by only 8 minutes (8′) of angle. Copernicus himself had never hoped to attain an accuracy greater than 10′, and the *Prussian Tables,* computed by Reinhold on the basis of Copernican methods, were off by as much as 5°. In 1609, before the application of telescopes to astronomy, 8′ was not a large angle; 8′ is just twice the minimum separation of two stars that the unaided average eye can distinguish as separate entities.

But Kepler was not to be satisfied by any approximation. He believed in the Copernican sun-centered system and he also believed in the accuracy of Tycho's observations. Thus, he wrote:

*The title indicates that this work is an *Astronomia nova,* a "new astronomy," in the sense of relating planetary motions to their causes so as to be a "celestial physics." In this particular aim Kepler was not successful—the first modern work to reveal the relationship between celestial motions and physical causes was Newton's *Principia* (1687).

> Since the divine goodness has given to us in Tycho Brahe a most careful observer, from whose observations the error of 8′ is shown in this calculation . . . it is right that we should with gratitude recognize and make use of this gift of God. . . . For if I could have treated 8′ of longitude as negligible I should have already corrected sufficiently the hypothesis . . . discovered in chapter xvi. But as they could not be neglected, these 8′ alone have led the way towards the complete reformation of astronomy, and have been made the subject-matter of a great part of this work.

Starting afresh, Kepler finally took the revolutionary step of rejecting circles altogether, trying an egg-shaped oval curve and eventually the ellipse. To appreciate how revolutionary this step actually was, recall that both Aristotle and Plato had insisted that planetary orbits had to be combined out of circles, and that this principle was a feature common to both Ptolemy's *Almagest* and Copernicus's *De revolutionibus.* Galileo, Kepler's friend, politely ignored the strange aberration. But the final victory was Kepler's. He not only got rid of innumerable circles, requiring but one oval curve per planet, but he made the system accurate and found a wholly new and unsuspected relation between the location of a planet and its orbital speed.

THE THREE LAWS

Kepler's problem was not only to determine the orbit of Mars, but at the same time to find the orbit of the earth. The reason is that our observations of Mars are made from the earth, which itself does not move uniformly in a perfect circle around the sun. Fortunately, however, the earth's orbit is almost circular. Kepler discarded Copernicus's idea that all planetary orbits should be centered on the mid-point of the earth's orbit. He discovered, instead, that *the orbit of each planet is in the shape of an ellipse with the sun located at one focus.* This principle is known as Kepler's first law.*

*In his book on Mars, Kepler first derives a general law of areas that is independent of any particular orbit. Only later, and by dint of enormous labor in calculation, does he invent the concept of an elliptical orbit, then finding that the orbit

Kepler's second law tells us about the speed with which a planet moves in its orbit. This law states that *in any equal time intervals, a line from the planet to the sun will sweep out equal areas.* Fig. 27 shows equal areas for three regions in a planetary orbit. Since the three shaded regions are of equal area, the planet moves most quickly when nearest to the sun and most slowly when farthest from the sun. This second law thus tells us at once that the apparent irregularity in the speed with which planets move in their orbits is a variation that is a function of a simple geometric condition.

The first and second laws plainly show how Kepler altered and simplified the Copernican system. But the third law, known also as the harmonic law, is even more interesting. It is called the harmonic law because its discoverer thought it demonstrated the true celestial harmonies. Kepler even entitled the book in which he announced it *The Harmony of the World* (1619). The third law states a relation between the periodic times in which the planets complete their orbits about the sun and their average distances from the sun. Let us make a table of the periodic times (T) and average distances (D). In this table and in the following text, the distances are given in astronomical units. One astronomical unit is, by definition, the mean distance from the earth to the sun. This table shows us that there is no simple relationship between D and T. Kepler therefore tried to see what would happen if he

	Mercury	*Venus*	*Earth*	*Mars*	*Jupiter*	*Saturn*
periodic time T (years)	0.24	0.615	1.00	1.88	11.86	29.457
mean distance from the sun D (astronomical units)	0.387	0.723	1.00	1.524	5.203	9.539

fits the observations re Mars. Some eighty years later, in the *Principia,* Newton deals with the area law first (props. 1–3) and only later (prop. 11) with the law of elliptical orbits.

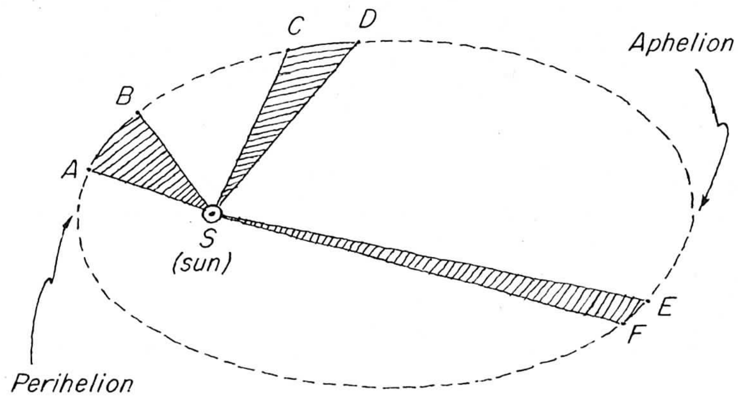

FIG. 27. Kepler's law of equal areas. Since a planet moves through the arcs AB, CD and EF in equal times (because the areas SAB, SCD, and SEF are equal), it travels fastest at perihelion, when nearest the sun, and slowest at aphelion, when farthest from the sun. The shape of this ellipse is that of a comet's orbit. Planetary ellipses are more nearly circular.

took the squares of these values, D^2 and T^2. These may be tabulated as follows (using today's values):

	Mercury	*Venus*	*Earth*	*Mars*	*Jupiter*	*Saturn*
T^2	0.058	0.378	1.00	3.53	141	867.7
D^2	0.150	0.523	1.00	2.323	27.071	90.993

There is still no relation discernible between D and T^2, or between D^2 and T, or even between D^2 and T^2. Any ordinary mortal would have given up at this point. Not Kepler! He was so convinced that these numbers must be related that he would never have given up. The next power is the cube. T^3 turns out to be of no use, but D^3 yields the following numbers. Note them and then turn back to the table of squares.

	Mercury	*Venus*	*Earth*	*Mars*	*Jupiter*	*Saturn*
D^3	0.058	0.378	1.00	3.54	141	867.9

Here then are the celestial harmonies, the third law, which states that the *squares of times of revolution of any two planets around the sun* (earth included) *are proportional to the cubes of their mean distances from the sun.*

In mathematical language, we may say that "T^2 is always proportional to D^3" or

$$\frac{D^3}{T^2}=K,$$

where K is a constant. If we choose as units for D and T the astronomical unit and the year, then K has the numerical value of unity. (But if the distance were measured in miles and time in seconds, the value of the constant K would not be unity.) Another way of expressing Kepler's third law is

$$\frac{D_1{}^3}{T_1{}^2}=\frac{D_2{}^3}{T_2{}^2}=\frac{D_3{}^3}{T_3{}^2}=\frac{D_4{}^3}{T_4{}^2}= \ldots\ldots\ldots = K$$

where D_1 and T_1, D_2 and T_2, . . . , are the respective distances and periods of any planet in the solar system.

To see how this law may be applied, let us suppose that a new planet were discovered at a mean distance of $4AU$ from the sun. What is its period of revolution? Kepler's third law tells us that the ratio D^3/T^2 for this new planet must be the same as the ratio $D_o{}^3/T_o{}^2$ for the earth. That is,

$$\frac{D^3}{T^2}=\frac{(1AU)^3}{(1^y)^2}.$$

Since $D = 4AU$,

$$\frac{(4AU)^3}{T^2}=\frac{(1AU)^3}{(1^y)^2},$$

$$\frac{64}{T^2}=\frac{1}{(1^y)^2}$$

$$T^2 = 64 \times (1^y)^2$$

$$T = 8^y.$$

The inverse problem may also be solved. What is the distance from the sun of a planet having a period of 125 years?

$$\frac{D^3}{T^2}=\frac{(1AU)^3}{(1^y)^2}$$

$$\frac{D^3}{(125^y)^2}=\frac{(1AU)^3}{(1^y)^2}$$

$$\frac{D^3}{125 \times 125}=\frac{(1AU)^3}{1}$$

$$D^3 = 25 \times 25 \times 25 \times (1AU)^3$$

$$D = 25AU.$$

Similar problems can be solved for any satellite system. The significance of this third law is that it is a law of necessity; that is, it states that it is impossible in any satellite system for satellites to move at just any speed or at any distance. Once the distance is chosen, the speed is determined. In our solar system this law implies that the sun provides the governing force that keeps the planets moving as they do. In no other way can we account for the fact that the speed is so precisely related to distance from the sun. Kepler thought that the action of the sun was, in part at least, magnetic. It was known in his day that a magnet attracts another magnet even though considerable distances separate them. The motion of one magnet produces motion in another. Kepler was aware that a physician of Queen Elizabeth, William Gilbert (1544–1603), had shown the earth to be a huge magnet. If all objects in the solar system are alike rather than different, as Galileo had shown and as the heliocentric system implies, why should not the sun and the other planets also be magnets like the earth?

Kepler's supposition, however tempting, does not lead directly to an explanation of why planets move in ellipses and sweep out equal areas in equal times. Nor does it tell us why the particular distance-period relation he found actually holds. Nor does it seem in any way related to such problems as the downward fall of bodies—according to the Galilean law of fall—on a stationary or on a moving earth, since the average rock or piece of wood is not magnetic. And yet we shall see that Newton, who eventually answered all these questions, based his discoveries on the laws found by Kepler and Galileo.

KEPLER VERSUS THE COPERNICANS

Why were Kepler's beautiful results not universally accepted by Copernicans? Between the time of their publication (I, II, 1609; III, 1619) and the publication of Newton's *Principia* in 1687, there are very few works that contain references to all three of Kepler's laws. Galileo, who had received copies of Kepler's books and who was certainly aware of the proposal of elliptic orbits, never referred in his scientific writings to any of the laws of Kepler, either to praise or to criticize them. In part, Galileo's reaction must have been Copernican, to stick to the belief in true circularity, implied in the very title of Copernicus's book: *On the Revolutions of the Celestial Spheres.* That work opened with a theorem: 1. *That the Universe is Spherical.* This is followed shortly after by a discussion of the topic, "That the motion of the heavenly bodies is uniform, circular, and perpetual, or composed of circular motions." The main line here is:

> Rotation is natural to a sphere and by that very act is its shape expressed. For here we deal with the simplest kind of body, wherein neither beginning nor end may be discerned nor, if it rotates ever in the same place, may the one be distinguished from the other. . . .
>
> We must conclude [despite any observed apparent irregularities, such as the retrogradations of planets] that the motions of these bodies are ever circular or compounded of circles. For the irregularities themselves are subject to a definite law and recur at stated times, and this could not happen if the motions were not circular, for a circle alone can thus restore the place of a body as it was. So with the Sun which, by a compounding of circular motions, brings ever again the changing days and nights and the four seasons of the year.

Kepler thus was acting in a most un-Copernican way by not assuming that the planetary orbits are either "circles" or "compounded of circles"; furthermore, he had come to his conclusion in part by reintroducing, at one stage of his thought, the one aspect of Ptolemaic astronomy to which Copernicus had most objected, the *equant.* In his astronomy, Kepler introduced a simple approximation to take the place of the law of areas. Kepler said that a line from any planet to the empty focus of its ellipse (Fig. 28) rotates uniformly, or that it does so very nearly. The empty focus, or the point about which such a line would rotate through equal angles in equal times, is the equant. (Incidentally, we may observe that this latter "discovery" of Kepler's is not true.)

From almost every point of view, the ellipses must have seemed objectionable. What kind of force could steer a planet along an

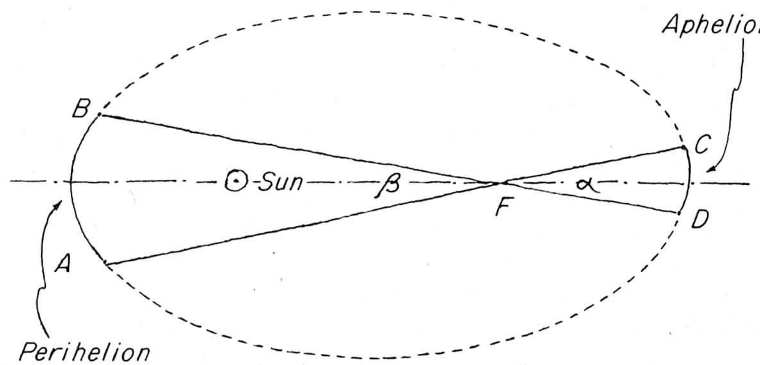

FIG. 28. Kepler's law of the equant. If a planet moves so that in equal times it sweeps out equal angles with respect to the empty focus at F, it will move through arcs $\widehat{AB}$ and $\widehat{CD}$ in the same time because the angles α and β are equal. According to this law, the planet moves faster along arc $\widehat{AB}$ (at perihelion) than along arc $\widehat{CD}$ (at aphelion) as the law of equal areas predicts. Nevertheless, this law is only a rough approximation. But in the seventeenth century, certain correction factors were added to this law to make it give more accurate results.

elliptical path with just the proper variation of speed demanded by the law of equal areas? We shall not reproduce Kepler's discussion of this point, but shall confine our attention to one aspect of it. Kepler supposed that some kind of force or emanation comes out of the sun and moves the planets. This force—it is sometimes called an *anima motrix*—does not spread out in all directions from the sun. Why should it? After all, its function is only to move the planets, and the planets all lie in, or very nearly in, a single plane, the plane of the ecliptic. Hence Kepler supposed that this *anima motrix* spread out only in the plane of the ecliptic. Kepler had discovered that light, which spreads in all directions from a luminous source, diminishes in its intensity as the inverse square of the distance; that is, if there is a certain intensity or brightness three feet away from a lamp, the brightness six feet away will be one-fourth as great because four is the square of two and the new distance is twice the old. In equation form,

$$\text{intensity} \propto \frac{1}{(\text{distance})^2}$$

But Kepler held that the solar force does not spread out in all directions according to the inverse-square law, as the solar light does, but only in the plane of the ecliptic according to a quite different law. It is from this doubly erroneous supposition that Kepler derived his law of equal areas—and he did so *before* he had found that the planetary orbits are ellipses! The difference between Kepler's procedure and what we would consider to be "logical" is that Kepler did *not* first find the actual path of Mars about the sun, and then compute its speed in terms of the area swept out by a line from the sun to Mars. This is but one example of the difficulty in following Kepler through his book on Mars.

THE KEPLERIAN ACHIEVEMENT

Galileo particularly disliked the idea that solar emanations or mysterious forces acting at a distance could affect the earth or any part of the earth. He not only rejected Kepler's suggestion that the sun might be the origin of an attractive force moving the earth and planets (on which the first two laws of Kepler were based), but he especially rejected Kepler's suggestion that a lunar force or emanation might be a cause of the tides. Thus he wrote:

> But among all the great men who have philosophized about this remarkable effect, I am more astonished at Kepler than at any other. Despite his open and acute mind, and though he has at his fingertips the motions attributed to the earth, he has nevertheless lent his ear and his assent to the moon's dominion over the waters, and to occult properties, and to such puerilities.

As to the harmonic law, or third law, we may ask with the voice of Galileo and his contemporaries, Is this science or numerology? Kepler already had committed himself publicly to the belief that the telescope should reveal not only the four satellites of Jupiter discovered by Galileo, but two of Mars and eight of Saturn. The reason for these particular numbers was that then the number of satellites per planet would increase according to a regular geometric sequence: 1 (for the earth), 2 (for Mars), 4 (for Jupiter), 8 (for Saturn). Was not Kepler's distance-period relation something of the same pure number-juggling rather than true science? And was not evidence for the generally nonscientific aspect of Kepler's whole book to be found in the way he tried to fit the numerical aspects of the planets' motions and locations into the questions posed by the table of contents for Book Five of his *Harmony of the World?*

1. Concerning the five regular solid figures.
2. On the kinship between them and the harmonic ratios.
3. Summary of astronomical doctrine necessary for contemplation of the celestial harmonies.
4. In what things pertaining to the planetary movements the simple harmonies have been expressed and that all those harmonies which are present in song are found in the heavens.
5. That the clefs of the musical scale, or pitches of the system, and the kinds of harmonies, the major and the minor, are expressed by certain movements.

6. That each musical Tone or Mode is in a certain way expressed by one of the planets.
7. That the counterpoints or universal harmonies of all the planets can exist and be different from one another.
8. That the four kinds of voice are expressed in the planets: soprano, alto, tenor, and bass.
9. Demonstration that in order to secure this harmonic arrangement, those very planetary eccentricities which any planet has as its own, and no others, had to be set up.
10. Epilogue concerning the sun, by way of very fertile conjectures.

Below are shown the "tunes" played by the planets in the Keplerian scheme.

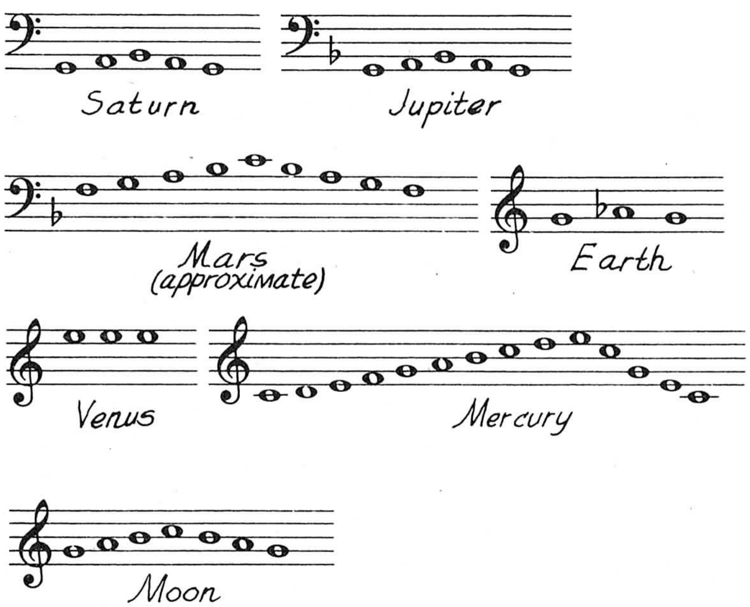

FIG. 29. Kepler's music of the planets, from his book *The Harmony of the World.* Small wonder a man of Galileo's stamp never bothered to read it!

Surely a man like Galileo would find it hard to consider such a book a serious contribution to celestial physics.

Kepler's last major book was an *Epitome of Copernican Astronomy,* completed for publication nine years before his death in 1630. In it he defended his departures from the original Copernican system. But what is of the most interest to us is that in this book, as in the *Harmony of the World* (1619), Kepler again proudly presented his earliest discovery concerning the five regular solids and the six planets. It was, he still maintained, the reason for the number of planets being six.

It must have been almost as much work to disentangle the three laws of Kepler from the rest of his writings as to remake the discoveries. Kepler deserves credit for having been the first scientist to recognize that the Copernican concept of the earth as a planet and Galileo's discoveries demanded that there be one physics—applying equally to the celestial objects and ordinary terrestrial bodies. But, alas, Kepler remained so enmeshed in Aristotelian physics that when he attempted to project a terrestrial physics into the heavens, the basis still came essentially from Aristotle. Thus the major aim of Keplerian physics remained unachieved, and the first workable physics for heaven and earth derived not from Kepler but from Galileo and attained its form under the magistral guidance of Isaac Newton.*

*Kepler did introduce the term "inertia" into the physics of motion, but the sense of Keplerian "inertia" was very different from the later (and present) significance of this term; see Supplement 8.